Let the following functions with domain $\mathbb{N}$, the set of natural numbers, and codomain $\mathbb{R}$, the set of real numbers.

1. $f_1(n) = \sum_{i=1}^{n}(7i+5)$,
2. $f_2(n) = \frac{4^{\log_2 n}}{n^2}$,
3. $f_3(n) = n\log_2(4^n)$,
4. $f_4(n) = n^{1/2}3^{2n+3}$,
5. $f_5(n) = 5 + \sum_{i=0}^{n} 3^i$,
6. $f_6(n) = 25 + 2^{3n+1}$.

a) Find, for each of these functions f_i, a function as simple as possible which is the estimate of Big-$\mathcal{O}$ as precise as possible for the function f_i.

b) We say that the function f grows less rapidly than the function g if f is in $\mathcal{O}(g)$ and g is not in $\mathcal{O}(f)$. It is said that the functions f and g grow at the same speed if f is in $\mathcal{O}(g)$ and g is in $\mathcal{O}(f)$. Arrange the functions f_1, f_2, f_3, f_4, f_5, f_6 from top to bottom depending on their growth rate (Big-$\mathcal{O}$). If functions are growing at the same speed, put their number on a same line.

Solution

a)

1. $f_1(n) \in \mathcal{O}(n^2)$,
2. $f_2(n) \in \mathcal{O}(1)$,
3. $f_3(n) \in \mathcal{O}(n^2)$,
4. $f_4(n) \in \mathcal{O}(n^{1/2}\ 9^n)$,
5. $f_5(n) \in \mathcal{O}(3^n)$,
6. $f_6(n) \in \mathcal{O}(8^n)$.

b)

1. $f_2(n)$,
2. $f_1(n)$ and $f_3(n)$,
3. $f_5(n)$,
4. $f_6(n)$,
5. $f_4(n)$.

Let the following functions with domain $\mathbb{N}$, the set of natural numbers, and codomain $\mathbb{R}$, the set of real numbers.

1. $f_1(n) = log_2(n^3 + 2n + 7)$,
2. $f_2(n) = 5^{2n+3}$.
3. $f_3(n) = \sqrt{n^{\frac{3}{7}} + n\log_3(7^n)}$,
4. $f_4(n) = 7n + \sum_{i=1}^{n}(3 + 2i)$,
5. $f_5(n) = \sum_{i=0}^{n} 7^i + 2i$,
6. $f_6(n) = \frac{125^{\log_5 n}}{n^2}$,

a) Find, for each of these functions f_i, a function as simple as possible which is the estimate of Big-$\mathcal{O}$ as precise as possible for the function f_i.

1. $f_1(n) \in \mathcal{O}(\log n)$,
2. $f_2(n) \in \mathcal{O}(25^n)$,
3. $f_3(n) \in \mathcal{O}(n)$,
4. $f_4(n) \in \mathcal{O}(n^2)$,
5. $f_5(n) \in \mathcal{O}(7^n)$,
6. $f_6(n) \in \mathcal{O}(n)$.

b) We say that the function f grows less rapidly than the function g if f is in $\mathcal{O}(g)$ and g is not in $\mathcal{O}(f)$. It is said that the functions f and g grow at the same speed if f is in $\mathcal{O}(g)$ and g is in $\mathcal{O}(f)$. Arrange the functions f_1, f_2, f_3, f_4, f_5, f_6 from top to bottom depending on their growth rate from the slower to the faster grow (Big-$\mathcal{O}$). If functions are growing at the same speed, put their number on a same line.

$f_1(n)$

$f_3(n)$ and $f_6(n)$

$f_4(n)$

$f_5(n)$

$f_2(n)$